PG – 217

IV Semester M.Sc. Degree Examination, June 2017
(CBCS) (16-17 and Onwards)
MATHEMATICS
M403TC : Computational Fluid Dynamics

Time : 3 Hours Max. Marks : 70

Instructions : 1) *All questions have **equal** marks.*
2) *Answer **any five** questions.*

1. a) Derive the Lax-Friedrich and Lax-Wendroff finite difference schemes for two-dimensional quasi linearized Euler's equation.

b) Solve the problem $U_t + U_x = 0$ with conditions

$$U(x, 0) = \begin{cases} \sin 2\pi x & , 0 \le x \le 1 \\ 0 & , 1 \le x \le 2 \end{cases}$$

$U(0, t) = 0$ using the forward time and backward space finite difference scheme with $\Delta x = 0.5$ and $\Delta t = 0.2$. Obtain the solution at second-time level. (7+7)

2. Obtain the quasi linearized equation $\frac{\partial v}{\partial t} + A_1 \frac{\partial v}{\partial x} + A_2 \frac{\partial v}{\partial y} + A_3 \frac{\partial v}{\partial z} = 0$ from inviscid Euler's equations. Where all the notations have usual meanings ? 14

3. Derive the explicit and implicit finite difference schemes for the Burger's equation and solve it with conditions

$$u(x, 0) = \frac{2\pi \sin \pi x}{2 + \cos \pi x}, \; 0 \le x \le 1$$

$$u(0, t) = 0 = u(1, t) \; , t \ge 0$$

Take $\Delta x = \frac{1}{4}$, $\lambda = \frac{1}{8}$. Obtain the solution at first time level. 14

P.T.O.

4. a) Define Node and Cell-centered control volume methods. Illustrate one of this with an example.

   b) Establish finite-volume via finite-difference method for the solution $U_{xx} + 6U_x + 5U = 0$. (7+7)

5. Obtain the cell-centered average scheme for the two-dimensional Euler equation

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0$$

where all the notations have usual meanings. 14

6. a) Discuss the main steps involved in the finite element method. Also discuss its advantages over the finite difference method.

   b) Obtain the Lagrange interpolation polynomial elements of approximating the solution. (7+7)

7. Solve the ordinary differential equation $y'' + 6y' + 5y(x) = x$ using the finite element method. 14

8. Obtain the global stiffness matrix that arises while solving the two dimensional Laplace equation $\Delta^2 u = 0$. 14
